
Math Graphs for the Visually Impaired: Audio Presentation of Elements of Mathematical Graphs

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ABSTRACT

The sense of sight takes a dominating role in learning mathematical graphs. Most visually impaired students drop out of mathematics because necessary content is inaccessible. Sonification and auditory graphs have been the primary methods of representing data through sound. However, the representation of mathematical elements of graphs is still unexplored. The experiments in this paper investigate optimal methods for representing mathematical elements of graphs with sound. The results indicate that the methods of design in this study are effective for describing mathematical elements of graphs, such as axes, quadrants and differentiability. These findings can help visually impaired learners to be more independent, and also facilitate further studies on assistive technology.

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KEYWORDS

Sonification; Visually Impaired; Assistive Technology; Auditory Presentation of Math Graphs.

CCS Concepts

•**Human-centered computing** → **Interaction techniques**; •**Human-centered computing** → **Auditory feedback**; •Social and professional topics → People with disabilities;

1 INTRODUCTION

Graphs are an important part of STEM areas, but are rarely accessible to those who are visually impaired or blind. Limited access to visual information has hindered the learning of mathematics and science for the visually impaired [1]. In this respect, sonification and auditory graphs have been primary tools for providing non-visual access to data using non-speech sound [2]. Sonification is the use of non-speech audio to convey information or perceptualize data [3], and an auditory graph is a sonified display that uses non-speech sound to explore quantitative data [4]. As Smith and Walker [5] stated, how best to design auditory displays of data is still an open question, so further studies are needed to find an optimal means of displaying data with sound. In addition, there is a lack of basic research into the effective ways of presenting the mathematical elements of graphs using audio.

The aim of this study is to find an optimal method for displaying mathematical elements of graphs with audio. The experiments presented in this paper mainly focused on three mathematical elements of graphs: (1) the x-axis, (2) quadrants and (3) differentiability. This paper provides the design and implementation of the experiments and the results of them.

2 RELATED WORK

Previous research has explored various ways of mapping data to sound. Flowers and Hauer [6] examined the non-visual presentation of data by combining various dimensions of sound, such as pitch and loudness. Neff and Pitt [7] suggested providing spatial audio to allow for a more accurate mapping of non-linear content. Existing studies have also examined various methods for providing natural and intuitive sonification. Walker and Mauney [8] investigated preferred polarities and scaling functions Harrar and Stockman [9] conducted some studies in which they compared continuous tones with discrete tones to convert data to sound, and Brown et al [10] presented a set of guidelines using stereo panning and speech information. Brewster [11] extended the studies of adding stereo panning and speech information to describe graphs using sound. Wanda [12] provided experimental evidence that sound can be used as an adjunct to improve sensitivity to the perception of data with noise.

3 BACKGROUND

Designing a method for the audio presentation of mathematical elements of graphs is a mostly unexplored field. The basic elements comprising the modern Cartesian coordinate system are the x- and y-axes, the coordinates of points and quadrants. On the basis of these basic elements, the experiments in this paper focused on the following mathematical elements of graphs: the x-axis, quadrants and differentiability. The sonification method used was the representation of the y-axis with pitch, and the representation of the x-axis with time. Stereo panning was also added to convey the x-axis position clearly.

The first experiment investigated methods of sonifying the x-axis. Previous research suggested providing contextual information by adding click sounds to assist blind users with exploring graphs [13]. The first experiment in this paper, however, focused on auditory representation of the

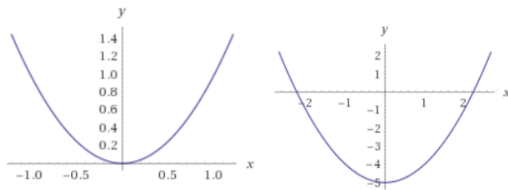


Figure 1: Relative positions of a graph and an x-axis which was sonified with parallel mode and serial mode.

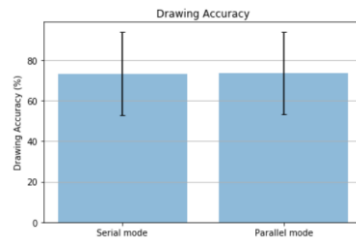


Figure 2(a)

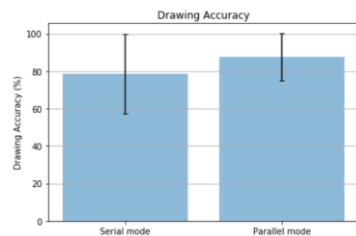


Figure 2(b)

Figure 2: Drawing accuracy of graphs. 2(a) shows the drawing accuracy of the first graph, with the x-axis at the bottom. 2(b) shows the drawing accuracy of the second graph, with the x-axis at the middle.

x-axis. In this experiment, participants were asked to estimate the position of a graph in relation to the x-axis.

The second experiment was carried out to examine a way to represent quadrants using varying timbres. The intersection of the x-axis and the y-axis creates four quadrants, and these quadrants are labeled counter-clockwise starting from the northeast quadrant. Despite few existing works on sonifying quadrants, devising a method for representing quadrants with sound is required. This second experiment addressed a sonification method for a closed curve, which requires a different approach from that used for a line graph.

The third experiment was conducted to examine whether non-differentiable points can be distinguished from differentiable points solely by differences of pitch. Evreinova et al [14] have noted that it would be hard to distinguish between these two types of points because their subjective contours are similar. In this experiment, TTS information was added to supplement this insufficiency of detailed information.

4. THE EXPERIMENT

4.1 Participants

32 participants took part in the study. The ages of the participants range from 17 to 45. All participants were legally blind, and included some participants with low vision. Each participant had a degree of knowledge regarding mathematical graphs, including basic concepts of axes, quadrants and differentiability. Before starting the experiments, an assistant read the consent forms to the blind participants and they accepted the agreement.

4.2 Method

This study used a within-subject design, using different versions of sound as independent variables. The first step of each task was to listen to an audio file representing graphs, after which the participants were asked to draw the graph as they perceived it immediately after listening. Each participant's drawing was then quantified for assessment based on a set of requirements which included overall shape, general features (i.e., maxima, minima and symmetry) and specific features (i.e., x-axis, quadrants and differentiability for each experiment). After the drawing task, they answered survey questions in the form of a Likert scale based on the NASA Task Load Index (TLX) [15]. Open-ended questions were also asked, focusing on overall experience and preference.

In the first experiment, the x-axis was sonified into two modes: parallel mode and serial mode. The parallel mode provided two sounds, one for a graph and the other for the x-axis at the same time, while the serial mode provided sound for x-axis first and sound for graph next.

During the second experiment, the participants heard two different modes of the same graph: one with quadrant information and the other without this information. The audio starts at point (1,0) of a Cartesian coordinate plane and proceeds counter-clockwise, in order of quadrant I, II, III and IV.

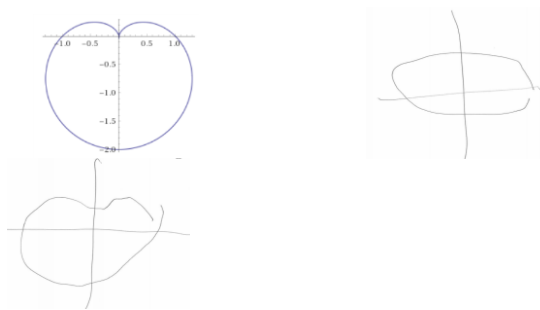


Figure 3(a) Figure 3(b) Figure 3(c)

Figure 3: Comparison of heart-shaped graphs. 3(a) shows the original heart shaped graph ($r=1-\sin(\theta)$) which was sonified. 3(b) shows a participant's drawing without quadrant information. 3(c) shows the same participant's drawing with quadrant information.

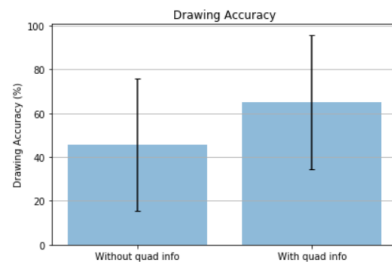


Figure 4: Drawing accuracy of graphs with and without quadrant information.

The timbre of sound changed to indicate the transition between quadrants, conveying which quadrant the audio displayed at a given moment.

In the third experiment, U-shaped graph which is a quadratic function ($y=x^2$) and V-shaped graph which is an absolute value function ($y=|x|$) were sonified into two modes, one with TTS speech information and the other without the information added.

5 RESULTS

5.1 Experiment1 – Sonification of the x-axis

[Fig. 1](#) shows the relative position of a graph and an x-axis. The results of the graph with the x-axis at the bottom showed no significant difference between the serial mode and the parallel mode ($t(31) = 0.1357, p > 0.05$). As for the workload, a paired T-test showed that the parallel mode was significantly more mentally and physically demanding than the serial mode ($t(31) = 3.9240, p = 0.0005$). This preference showed similar scores, in which 62.5% of the participants preferred serial mode to parallel mode. The drawing accuracy for the graph with the x-axis at the middle showed that participants had higher drawing accuracy in the parallel mode (87.71%) compared to the serial mode (78.44%) ($t(31) = 2.2577, p < 0.05$). The workload when carrying out the drawing of graphs showed no significant difference ($t(31) = 0.4557, p > 0.05$). 70.0% of the participants preferred parallel mode to serial mode, which is opposite of the previous graph with the x-axis at the bottom. [Fig. 2](#) shows the drawing accuracy of two graphs for each mode.

The reasons for this conflicting result in terms of the preference were demonstrated in the following quotes from post-interviews.

“The serial mode was more convenient for me because distinguishing between the sound for a graph and the sound for the x-axis was a bit confusing with parallel mode. However, it was easy to find the intersection points with parallel mode.” (P.e_3)

“The serial mode was better for drawing graphs. I can only draw a graph serially, not in parallel.” (P.e_17)

Serial mode was preferred for its lower cognitive load and ease of carrying out the drawing task, though parallel mode was better for recognizing intersection points and drawing accurately. In parallel mode, intersection points could be indicated when two sounds merge into a single sound, whereas they were hard to identify in serial mode.

Some parts of additional comments from participants are shown below:

“Parallel mode requires more concentration.” (P.e_30)

“In serial mode, I should remember the pitch of the x-axis first and then listen to sound of a graph. Parallel mode however enables me to compare two sounds with ease.” (P.e_32)

Parallel mode obviously imposes additional load, however it would be more efficient for learning in terms of recognizing intersection points and comparing the relative change of pitches.



Figure 5. (a) Figure 5. (b) Figure 5. (c)

Figure 5: Comparison of U-shaped graphs. 5(a) shows the original U-shaped graph ($y=x^2$) which was sonified. 5(b) shows a participant's drawing without speech information. 5(c) shows the same participant's drawing with speech information.



Figure 6. (a) Figure 6. (b) Figure 6. (c)

Figure 6: Comparison of V-shaped graphs. 6(a) shows the original V-shaped graph ($y=|x|$) which was sonified. 6(b) shows a participant's drawing without speech information. 6(c) shows the same participant's drawing with speech information.

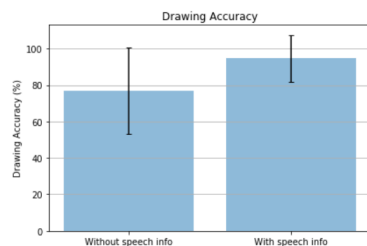


Figure 7: Drawing accuracy of graphs with and without speech information.

5.2 Experiment2 – Sonification of quadrants

The graph on Fig. 3 is a closed curve which was sonified in this experiment. Fig. 3 represents three graphs: original graph which was sonified, a participant's drawing performed with and without quadrant information. Drawing accuracy differed significantly depending on the presence of the quadrant information. The addition of quadrant information showed 65.10% of drawing accuracy, while the absence of quadrant information resulted in 45.52% of drawing accuracy ($t(31) = 4.2727$, $p = 0.0001$). Fig. 4 shows the drawing accuracy of graphs with and without the quadrant information. The cognitive load showed no significant difference ($t(31) = 1.3652$, $p > 0.05$). 87.5% of participants preferred the audio with quadrant information. This interpretation was strengthened by the following quotes from post-interviews: "I couldn't even guess which quadrant the graph falls in at that moment without the quadrants information." (P.e_17)

"The change of timbre indicating change of quadrants was helpful, but it puts a little pressure on me because I had to remember four kinds of timbres." (P.e_3)

Most participants drew the graph represented without quadrant information as either a circle or a shape of wave such as a sine graph, which indicates that it is difficult to convey spatial sense and details of graphs without using quadrant information.

5.3 Experiment3 – Sonification of differentiability

Fig. 5 and Fig. 6 show the original graph which was sonified and the drawings of the same participant with and without speech information. The drawing accuracy was significantly different between drawings with and without speech information ($t(31) = 4.3481$, $p = 0.0001$). Fig. 7 shows the drawing accuracy of graphs with and without speech information. The drawing accuracy with speech information was 94.58% and the accuracy without speech information was 76.88%. All of the participants preferred the addition of speech information for recognizing the differentiability, and the reasons for this preference are demonstrated in the following quotes from the interviews.

"I recognized that there exists a difference between two sounds, however I could not pinpoint exactly what the difference between two graphs is. Speech information clarified the difference without doubt." (P.e_32)

"I think I can figure out almost every shape of graphs with the proper combination of speech information and [sonified] graph sounds." (P.e_33)

As for the mode without speech information, all of the participants rarely found it possible to distinguish between two graphs solely depending on the difference of pitch, and they drew both of graphs as U-shaped curves, which implies that an addition of speech information should not be completely ruled out, especially when a subtle distinction exists. Comments for the speech information were mostly positive, praising the clarity of description for the mathematical elements.

6. DISCUSSION

The experiments in this paper explored various methods for audio representation of mathematical elements of graphs and the results suggest a set of guidelines for designing auditory displays of mathematical elements of graphs. Firstly, it is recommended to display the x-axis and a graph separately (i.e., serially).

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However, if intersection points exist between the x-axis and graph, then using parallel mode is desirable for recognizing intersection points. Secondly, using changes of timbre is an efficient method for indicating transitions between quadrants and it is especially required when sonifying graphs across four quadrants. Finally, the addition of speech information is required for recognizing differentiability. These results can be used to find an optimal method for representing mathematical elements of graphs, which is clearly needed for the visually impaired to access to mathematical graphs and to become independent learners. Designers and researchers of auditory graphs for the visually impaired can also use these findings to improve their products or designs.

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